Problema 9.3.1.

Pentru următoarele funcţii booleene de trei variabile, date prin intermediul tabelelor de valori, scrieţi cele două forme canonice: *conjunctivă* (FCC) şi *disjunctivă* (FCD). Simplificaţi funcţiile utilizând diagrame Veitch.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *x* | *y* | *z* | *f* | *mintermi/Maxtermi* |
| 0 | 0 | 0 | 1 | *m*0=*x*0*y*0*z*0=¯ *x* ¯ *y* ¯ *z* |
| 0 | 0 | 1 | 1 | *m*1=*x*0*y*0*z*1=¯*x* ¯ *y z* |
| 0 | 1 | 0 | 0 | *M*2= == |
| 0 | 1 | 1 | 0 | *M*3=== |
| 1 | 0 | 0 | 0 | *M*4=== |
| 1 | 0 | 1 | 1 | *m*5==*x*1*y*0*z*1= *x* ¯ *y z* |
| 1 | 1 | 0 | 0 | *M*6=== |
| 1 | 1 | 1 | 1 | *m*7==*x*1*y*1*z*1= *x y z* |

FCC(*f* ) = *M*2 ∧ *M*3 ∧ *M*4 ∧ *M*6 = () ∧ () ∧ () ∧ ()

*Mi=*

FCD(*f* ) = *m*0 ∨ *m*1 ∨ *m*5 ∨ *m*7 = ¯ *x* ¯ *y* ¯ *z* ∨¯*x* ¯ *y z* ∨ *x* ¯ *y z* ∨ *x y z*

1. Factorizarea:

Diagrama Veitch:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *x* | | | ¯ *x* | |
| *y* | *m*7 |  |  |  |
| ¯ *y* | *m*5 |  | *m*0 | *m*1 |
| *z* | ¯ *z* | | *z* |

*max*1= *m*0 ∨ *m*1= ¯ *x* ¯ *y*

*max*2= *m*7 ∨ *m*5= *x* *z*

*max*3= *m*5 ∨ *m*1= ¯ *y z*

M(*f* ) = { *max*1, *max*2, *max*3}

1. Mulțimea monoamelor centrale

C(*f* ) = { *max*1, *max*2}

M(*f* ) ≠ C(*f* ) , C(*f* ) ≠ ∅ ⇒ cazul II *g* (*x*,*y*,*z*)= *max*1∨ *max*2

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *x* | | | ¯ *x* | |
| *y* | *m*7 |  |  |  |
| ¯ *y* | *m*5 |  | *m*0 | *m*1 |
| *z* | ¯ *z* | | *z* |

Se observă că *Sg*= *Sf* ⇒ avem o singură formă simplificată, *f’*(*x*,*y*,*z*)= *max*1∨ *max*2 = ¯ *x* ¯ *y* ∨ ¯ *y z*

Problema 9.3.2.

Simplificaţi următoarele funcţii booleene de patru variabile, date prin formele canonice disjunctive, utilizând diagrame Veitch:

*m*15 *m*14 *m*6 *m*8 *m*0 *m*2 *m*9 *m*1 *m*3

*f*(*x*1*,x*2*,x*3*,x*4)*=* *x*1*x*2*x*3*x*4 ∨ *x*1*x*2*x*3¯*x*4 ∨ ¯*x*1*x*2*x*3¯*x*4 ∨ *x*1¯*x*2¯*x*3¯*x*4 ∨ ¯*x*1¯*x*2¯*x*3¯*x*4 ∨ ¯*x*1¯*x*2*x*3¯*x*4 ∨ *x*1¯*x*2¯*x*3*x*4 ∨ ¯*x*1¯*x*2¯*x*3*x*4 ∨ ¯*x*1¯*x*2*x*3*x*4;

Factorizarea:

Diagrama Veitch:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *x*1 | | | ¯ *x*1 | | |
| *x*2 | *m*15 |  |  |  | *x*4 |
| *m*14 |  |  | *m*6 | ¯*x*4 |
| ¯ *x*2 |  | *m*8 | *m*0 | *m*2 |
|  | *m*9 | *m*1 | *m*3 | *x*4 |
| *x*3 | ¯ *x*3 | | *x*3 |

*max*1= *m*0 ∨ *m*1∨ *m*2∨ *m*3= ¯ *x*1¯ *x*2

*max*2= *m*14 ∨ *m*15= *x*1*x*2*x*3

*max*3= *m*8 ∨ *m*9∨ *m*0∨ *m*1= ¯ *x*2¯ *x*3

*max*4= *m*6 ∨ *m*14= *x*2*x*3¯*x*4

*max*5= *m*6 ∨ *m*2= ¯*x*1*x*3¯*x*4

M(*f* ) = { *max*1, *max*2, *max*3, *max*4, *max*5}

Mulțimea monoamelor centrale

C(*f* ) = { *max*3, *max*2, *max*1}

M(*f* ) ≠ C(*f* ) , C(*f* ) ≠ ∅ ⇒ cazul II *g* (*x*1*,x*2*,x*3*,x*4)= *max*3∨ *max*2∨ *max*1

*h*1(*x*1*,x*2*,x*3*,x*4)= *max*4

*h*2(*x*1*,x*2*,x*3*,x*4)= *max*5

sunt 2 forme simplificate:

*f* 1‘(*x*1*,x*2*,x*3*,x*4)*= g* (*x*1*,x*2*,x*3*,x*4) ∨ *h*1(*x*1*,x*2*,x*3*,x*4)= *max*3∨ *max*2∨ *max*1∨ *max*4= ¯ *x*2¯ *x*3∨ *x*1*x*2*x*3∨ ¯ *x*1¯ *x*2∨ *x*2*x*3¯*x*4

*f* 2‘(*x*1*,x*2*,x*3*,x*4)*= g* (*x*1*,x*2*,x*3*,x*4) ∨ *h*2(*x*1*,x*2*,x*3*,x*4)= *max*3∨ *max*2∨ *max*1∨ *max*5= ¯ *x*2¯ *x*3∨ *x*1*x*2*x*3∨ ¯ *x*1¯ *x*2∨ ¯*x*1*x*3¯*x*4

Problema 9.3.3.

Simplificaţi următoarele funcţii booleene de trei variabile, date prin mintermii expresiilor, utilizând diagrame Karnaugh:

1. *f*2(*x*1*,x*2*,x*3)*=* *m*1 ∨ *m*2 ∨ *m*4 ∨ *m*5 ∨ *m*6 ∨ *m*7;

Factorizarea:

Diagrama Karnaugh:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *x*2*x*3  *x*1 | 00 | 01 | 11 | 10 |
| 0 |  | *m*1 |  | *m*2 |
| 1 | *m*4 | *m*5 | *m*7 | *m*6 |

*max*1= *m*1∨ *m*5 = *x*20 *x*31=¯*x*2*x*3

*max*2= *m*2∨ *m*6 = *x*21 *x*30=*x*2¯*x*3

*max*3= *m*4∨ *m*5∨ *m*7 ∨ *m*6 = *x*11=*x*1

M(*f* ) = { *max*1, *max*2, *max*3 }

Mulțimea monoamelor centrale

C(*f* ) = { *max*1, *max*2, *max*3 }

M(*f* ) = C(*f* ) ⇒ cazul I

*f*‘(*x*1*,x*2*,x*3)*= max*1∨ *max*2∨ *max*3=¯*x*2*x*3∨ *x*2¯*x*3∨ *x*1

Problema 9.3.7.

Simplificaţi următoarele funcţii booleene de patru variabile utilizând diagrama Karnaugh:

1. *f*(*x*1*,x*2*,x*3*,x*4)*=* *x*1*x*2 ∨ *x*1¯*x*2¯*x*3*x*4 ∨ ¯*x*1¯*x*2*x*4 ∨ ¯*x*1*x*3 ∨ *x*2*x*3;

*m*15 *m*13 *m*14 *m*12

*x*1*x*2= *x*1*x*2*x*3*x*4∨ *x*1*x*2¯*x*3*x*4 ∨ *x*1*x*2*x*3¯*x*4∨ *x*1*x*2¯*x*3¯*x*4

*m*9

*x*1¯*x*2¯*x*3*x*4

*m*3 *m*1

¯*x*1¯*x*2*x*4=¯*x*1¯*x*2*x*3*x*4∨¯*x*1¯*x*2¯*x*3*x*4

*m*7 *m*3 *m*6 *m*2

¯*x*1*x*3=¯*x*1*x*2*x*3*x*4∨¯*x*1¯*x*2*x*3*x*4∨¯*x*1*x*2*x*3¯*x*4∨¯*x*1¯*x*2*x*3¯*x*4

*m*15 *m*7 *m*14 *m*6

*x*2*x*3= *x*1*x*2*x*3*x*4∨¯*x*1*x*2*x*3*x*4∨ *x*1*x*2*x*3¯*x*4∨¯*x*1*x*2*x*3¯*x*4

Factorizarea:

Diagrama Karnaugh:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *x*3*x*4  *x*1*x*2 | 00 | 01 | 11 | 10 |
| 00 |  | *m*1 | *m*3 | *m*2 |
| 01 |  |  | *m*7 | *m*6 |
| 11 | *m*12 | *m*13 | *m*15 | *m*14 |
| 10 |  | *m*9 |  |  |

*max*1= *m*12∨ *m*13∨ *m*14∨ *m*15 = *x*1*x*2

*max*2= *m*3∨ *m*2∨ *m*7∨ *m*6 =¯*x*1*x*3

*max*3= *m*1∨ *m*9 =¯*x*2¯*x*3*x*4

*max*4= *m*1∨ *m*3 =¯*x*1¯*x*2*x*4

*max*5= *m*9∨ *m*13 = *x*1¯*x*3*x*4

*max*6= *m*15∨ *m*14∨ *m*7∨ *m*6 = *x*2*x*3

M(*f* ) = { *max*1, *max*2, *max*3, *max*4, *max*5, *max*6}

Mulțimea monoamelor centrale

C(*f* ) = { *max*1, *max*2 }

M(*f* ) ≠ C(*f* ) , C(*f* ) ≠ ∅ ⇒ cazul II *g* (*x*1*,x*2*,x*3*,x*4)= *max*1∨ *max*2

*h* (*x*1*,x*2*,x*3*,x*4)= *max*3

este o singură formă simplificată:

*f*‘(*x*1*,x*2*,x*3*,x*4)*= g* (*x*1*,x*2*,x*3*,x*4) ∨ *h* (*x*1*,x*2*,x*3*,x*4)= *max*1∨ *max*2∨ *max*3 = *x*1*x*2∨¯*x*1*x*3∨¯*x*2¯*x*3*x*4

Problema 9.3.4.

Utilizând metoda lui Quine simplificaţi următoarele funcţii booleene de trei variabile:

|  |  |
| --- | --- |
|  | 2. *f*2(*x*1*,x*2*,x*3)*=* *x*3(*x*1∨ *x*2) ∨ (¯*x*2↓*x*3); |
|  |  |

Problema 9.3.5.

Simplificaţi următoarele funcţii booleene de patru variabile date prin valorile de 1, utilizând metoda lui Quine:

1. *f*2(1,1,0,1)=1, *f*2(0,1,0,1)=1, *f*2(0,1,0,0)=1, *f*2(0,0,0,0)=1, *f*2(0,0,1,0)=1, *f*2(1,0,1,1)=1, *f*2(1,0,0,1)=1, *f*2(0,0,1,1)=1;

Problema 9.3.6.

Simplificaţi următoarele funcţii booleene de trei variabile date prin zerourile acestora, utilizând metoda lui Quine:

1. *f*2(0,0,0) = *f*2(0,0,1) = *f*2(1,1,1) = 0;

S*f* ={(0,1,1),(1,0,1),(0,1,0),(1,0,0),(1,1,0)}

Tabelul de factorizare:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Grupul |  | *x*1 | *x*2 | *x*3 |  |